Correspondence

Tuning Range of the Backward Traveling-Wave Parametric Amplifier*

Experimental results obtained with backward traveling-wave parametric amplifiers[1,2] indicate that their tuning range is greater than that predicted when TEM propagation is assumed.[1] An analytical and graphical examination of the possible tuning range when non-TEM transmission lines are considered is given below. This examination shows how the tuning range may be optimized, and indicates what the maximum range would be using realizable transmission lines.

Consider first a small perturbation which transforms the linear transmission line characteristics to the form

$$\omega_\alpha = v_\alpha \beta_\alpha - c_\alpha \beta_\alpha{}^2$$

$$(\alpha = p, i, \text{ or } s \text{ for pump, idle or signal.}) \quad (1)$$

The conditions for parametric amplification are[3]

$$\omega_p = \omega_i + \omega_s \quad (2)$$

$$\beta_p = \beta_i - \beta_s. \quad (3)$$

If one takes $4c_\alpha \omega_\alpha / v_\alpha{}^2 \ll 1$, one finds upon combining (1)–(3) that

$$\omega_s = \frac{\omega_p}{2}(1 - r + r\omega_p \epsilon) \quad (4)$$

$$\omega_i = \frac{\omega_p}{2}(1 + r - r\omega_p \epsilon), \quad (5)$$

where $r = v_s/v_p$, $\epsilon = c_s/v_s{}^2 - c_p/v_p{}^2$, and signal and idle propagate in the same transmission line. It is seen that both ω_s and ω_i change as ω_p is varied. Limitations on the tuning range are imposed by the condition that signal, idle, and pump frequency bands must be separated. For TEM lines, or in the more general case $\epsilon = 0$, one must have $r = r_0 = 0.236$, in order to achieve the maximum tuning range $(\omega_{sH}/\omega_{sL})_0 = 1.618$ (ω_{sH} and ω_{sL} are the highest and lowest signal frequencies).[1] For the perturbation, we let

$$r = r_0 + r_1. \quad (6)$$

Then, to first order, one finds

$$\frac{\omega_{sH}}{\omega_{sL}} = \left(\frac{\omega_{sH}}{\omega_{sL}}\right)_0 + \left[\frac{r_0}{1 + r_0} + \frac{2r_0{}^2}{(2 + r_0)(1 + r_0)^2}\right]\omega_{pH_0}\epsilon$$

$$= 1.618 + 0.224\omega_{pH_0}\epsilon \quad (7)$$

for

$$r = r_0 + \frac{2r_0}{2 + r_0}\omega_{pH_0}\epsilon = 0.236 + 0.211\omega_{pH_0}\epsilon. \quad (8)$$

Thus, the tuning range is increased for $\epsilon > 0$.

If instead of (1), one considers the case of a TEM signal-idle line and a high-pass waveguide pump circuit, a perturbation calculation shows that the maximum tuning range must be reduced. It is also reduced when the transmission line characteristics are approximated by

$$\omega_\alpha - \omega_{c\alpha} = v_\alpha \beta_\alpha, \quad (9)$$

where ω_c is the cutoff frequency of the transmission line.

Although the above analytical approach is valuable in predicting how the tuning range will be affected, it cannot yield quantitative results since it utilizes only an approximation to ω-β characteristics actually encountered in experiment. In order to ascertain what the maximum tuning range could be and what value of r to choose in the amplifier design, one must resort to a graphical technique which is, however, quite simple. Fig. 1 illustrates this procedure, which is as follows:

1) Draw the signal-idle circuit ω-β characteristic (assume it is $\omega_s = v_s \sin \beta_s/2$).
2) Draw an arbitrary pump-circuit ω-β characteristic (assume $\omega_p = v_p\beta_p$ and try $r = 0.4$).
3) Choose the cutoff of the low-pass signal line as ω_{iH}. This will maximize the tuning range, since we know from the analysis that the ω-β nonlinearity is favorable here.
4) Find ω_{sH} such that $(\omega_{pH}, \beta_{pH})$ is a point falling on the pump characteristic. This step is a rapid trial-and-error procedure.
5) The ω_{sH} point is also taken to be ω_{iL} and 4) is repeated for the low end of the tuning range.
6) Find $\omega_{sH}/\omega_{sL} = 1.89$; note that $\omega_{pL} > \omega_{iH}$, indicating that $r = 0.4$ is not optimum.
7) Redraw the pump characteristic with increased r (try $r = 0.5$).
8) Repeat 3)–5) and find $\omega_{sH}/\omega_{sL} = 2.48$; note that $\omega_{pL} < \omega_{iH}$, which may not be permitted.
9) Redraw the pump characteristic with decreased r (try $r = 0.475$).
10) Find $\omega_{sH}/\omega_{sL} = 2.31$ and $\omega_{pL} = \omega_{iH}$ indicating that this last choice of r is optimum.

If the signal circuit had had a low-frequency cutoff, as shown by the dashed line in Fig. 1, the optimum r would have been $r = 0.5$, but the tuning range would have been only 2.07.

Finally, it is of interest to apply the graphical technique to the actual experimental configuration used by Breitzer and Sard.[1] This is illustrated in Fig. 2. The question asked is: what are the expected signal and idle ranges when $r = \frac{1}{3}$ and the pump frequency is swept from 4.5 to 7.0 Mc? Table I illustrates the excellent agreement between the graphical predictions and the experimental results as computed by Breitzer and Sard.[4]

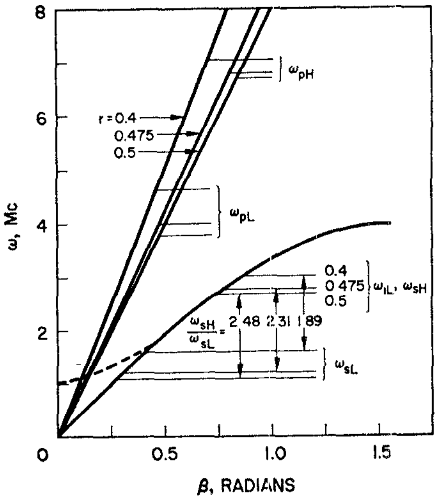

Fig. 1—Illustration of graphical procedure.

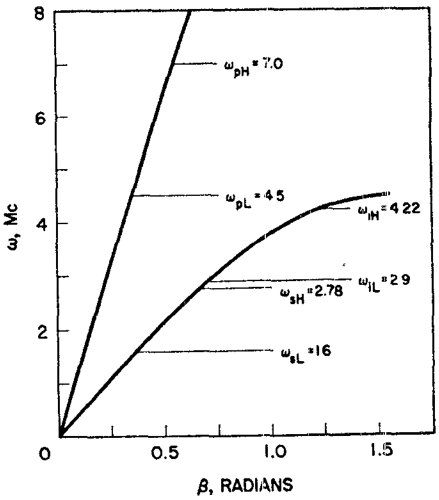

Fig. 2—Graphical construction corresponding to Breitzer and Sard's BWPA.[1]

TABLE I

	Graphical Predictions	Experimental Results	Linear Theory
ω_{sH}/ω_{sL}	1.73	1.70	1.55
ω_{iH}/ω_{iL}	1.46	1.48	1.55

In conclusion, it has been shown that tuning ranges considerably larger than 2:1 should be obtained when a proper choice of transmission-line characteristics is made. The predicted tuning range can then be expected to correspond closely with the experimental results.

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[1] D. I. Breitzer and E. W. Sard, "Low frequency prototype backward-wave reactance amplifier," *Microwave J.*, vol. 2, pp. 34–37; August, 1959.

[2] H. Hsu, "Backward Traveling-Wave Parametric Amplifiers," presented at Solid State Circuits Conf., Philadelphia, Pa.; February 10–12, 1960.

[3] P. K. Tien, "Parametric amplification and frequency mixing in propagating circuits," *J. Appl. Phys.*, vol. 29, pp. 1347–1357; September, 1958.

[4] Breitzer and Sard, *op. cit.*, Fig. 3.